Peer Effects and Incentives*

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Abstract

In a multi-agent setting, individuals often compare own performance with that of their peers. These comparisons influence agents' incentives and lead to a noncooperative game, even if the agents have to complete independent tasks. I show that depending on the interplay of the peer effects, agents' efforts are either strategic complements or strategic substitutes, but the Informativeness Principle always applies. I solve for the optimal monetary incentives that complement the peer effects. In case of limited liability, the principal may prefer to implement inefficiently large efforts although agents earn positive rents that increase in the respective agent's effort level.

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"“pure” peer effects refer to a situation where workers work, side by side, for the same firm but do not interact in any way (except that they observe each others' work activity)"

(Charness and Kuhn 2011, p. 255)

1 Introduction

Real agents typically compare their economic outcomes with one another and have – to some extent – so-called social or other-regarding preferences. Theories on these preferences assume that the agents compare their incomes or payoffs (Fehr and Schmidt 1999, Bolton and Ockenfels 2000, Charness and Rabin 2002, Fershtman et al. 2003). However, in practice, agents often do not know the *incomes* or *payoffs* of their peers but mutually observe each other's *performance*.[1] Empirical studies show that this performance information strongly influences workers' effort choices (Falk and Ichino 2006, Mas and Moretti 2009, Gächter et al. 2013, Georganas et al. 2013). From a theoretical perspective, such peer effects have to be kept in mind by a principal when designing optimal incentives in a multi-agent setting.

In this note, I analyze the interplay of incentives arising from peer effects and corresponding optimal monetary incentives. Following the observations by Sheremeta (2010) and Dohmen et al. (2011), I model peer effects as additional utility or disutility arising from the mere fact of outperforming one's peer or being outperformed by the peer, respectively. In the first case, we can also speak of positive externalities and in the latter case of negative externalities that an agent receives from comparing own performance with peer performance. The analysis shows that, even if the agents have to complete independent tasks, peer effects will lead to a game between the agents, which the principal has to anticipate when designing optimal incentives. Depending

[1] Charness et al. (2014) report that, even if agents lack this performance information and if their compensation does not depend on their peers' output, firms nevertheless provide agents with relative performance information.

on the magnitude of the externalities arising from peer effects, agents' efforts are either strategic substitutes or strategic complements.

First, I consider a situation in which the principal does not face restrictions on the optimal contract choice. If peer effects are quite strong for one agent and rather weak for the other, the principal will implement excessive effort by the former agent and little effort by the latter. Otherwise, the principal is confronted with a kind of coordination problem: (i) If negative externalities dominate positive ones, the principal will prefer either excessive efforts by both agents or little efforts by both agents to prevent that one of the agents outperforms his peer. (ii) If positive externalities dominate negative ones, the principal will prefer excessive effort by one of the agents and little effort by the other to generate a net gain in terms of externalities.

If the principal's contracting space is restricted to non-negative wages (limited liability) and the agents earn positive rents, peer effects will unambiguously benefit the principal. He particularly profits from large negative externalities, which make efforts be strategic complements so that incentives for one agent spill over to his peer. Sufficiently large negative externalities can even make the principal implement *inefficiently large* efforts although agents earn positive rents that increase in the respective agent's effort level. Finally, if the principal can choose between agents moving simultaneously or sequentially, he will strictly prefer a sequential-move setting.

Peer effects crucially differ from preferences based on relative income, like inequity aversion introduced by Fehr and Schmidt (1999). As Englmaier and Wambach (2010) show, if inequity aversion is sufficiently strong, the principal will prefer team compensation despite independent tasks to eliminate inequity costs, which contradicts the Sufficient Statistic Theorem or Informativeness Principle of Holmstrom (1979). I show that in case of peer effects, however, the Informativeness Principle always applies and optimal incentives can be solely based on individual performance although agents' efforts are mutually influenced by positive and negative externalities.

As peer effects lead to a mutual comparison of relative performance among the agents, there are parallels to the tournament literature (e.g., Moldovanu and Sela 2001). However, in a tournament relative performance comparison arises from the principal's incentive scheme which gives the best performer the highest monetary reward, the second best performer the second highest reward and so on. From a contract theoretic perspective, there exist two arguments why the set of possible contracts is restricted to the class of tournament contracts – either the agents' tasks are related so that relative performance evaluation is optimal against the background of the Informativeness Principle (Holmstrom 1982), or agents' individual performance measures are non-contractible so that a tournament is used by the principal as a self-commitment device (Malcomson 1984).

There exist other papers that also address peer effects, but do this in a completely different way. For example, Winter (2010) considers a multi-agent setting in which one agent may observe the effort choice of another agent and, therefore, can make the own effort choice contingent on that observation. Such peer effects are relevant because the agents' tasks are not independent and only joint team outcome serves as a contractible performance measure for each agent. The model further differs from my setting as effort choice and team output are assumed to be binary.

The only preliminary work that is more closely related to my setting is Kräkel (2008a, 2008b). Kräkel (2008a) considers a tournament setting that is supplemented by emotional effects based on the heterogeneity of agents. If a weak agent beats a strong opponent, the former agent will receive an extra utility from feeling pride whereas the latter one will receive a disutility from being ashamed. Kräkel (2008b) considers workers that are rewarded according to linear piece-rate contracts. The workers may feel two kinds of emotions, either from being successful or failing concerning the own task, or from being more or less successful than the co-worker. The latter kind of emotions are similar to the peer effects considered in this paper. However, the paper only considers the case of unlimited liability. Moreover, it does neither address the existence of strategic complements and substitutes nor the validity of the Informativeness Principle. Kräkel (2008b) shows that the firm benefits (suffers) from positive (negative) emotions of the workers, which immediately follows from the workers' binding participation constraints under the optimal piece-rate contract.

2 The Basic Model

I consider a situation in which a principal, P, must hire two agents, A and B, in order to run a business.[2] All parties are risk-neutral. A and B have zero reservation values. When working for P, agent i ($i = A, B$) exerts effort $e_i \in [0, 1]$ to generate a return $R_i \in \{0, R\}$ for P. As an example, we can think of a sales agent that either acquires a new customer or not. Alternatively, imagine a researcher that either succeeds in publishing in one of the top journals or not. With probability $\Pr(R_i = R|e_i) = e_i$ agent i is successful and generates the high return $R > 0$ and with probability $\Pr(R_i = 0|e_i) = 1 - e_i$ the agent fails and realizes 0. The realization of R_i is verifiable, but P cannot observe e_i (moral hazard). Effort e_i entails costs on agent i being described by the function c_i with the usual technical characteristics $c_i'(e_i), c_i''(e_i) > 0$, $c_i'''(e_i) \geq 0$ for $e_i > 0$, and $c_i(0) = c_i'(0) = 0$, $\lim_{e_i \to 1} c_i'(e_i) = \infty$.

I deviate from the textbook moral-hazard model by assuming peer effects between the agents, who both observe R_A and R_B. If agent i is more successful than agent j, there will be a negative and a positive externality: i's payoff is enlarged by $\alpha_i > 0$, whereas j's payoff is reduced by $\beta_j > 0$. If both agents' performance is the same (i.e., $R_A = R_B$), there will be no externali-

[2] Technically, P's reservation value is minus infinity so that P always wants to induce A and B to accept the offered contracts. The main assumptions follow the single-agent setting of Schmitz (2005) and Ohlendorf and Schmitz (2012). Externalities are modeled similar to emotional effects considered by Kräkel (2008a, 2008b).

ties. Let $\Psi := \beta_A + \beta_B - \alpha_A - \alpha_B \neq 0$ so that externalities do not cancel each other out, and $\inf_{e_A,e_B} c''_A(e_A)\, c''_B(e_B) > \Psi^2$ to guarantee that P's objective function is well-behaved.[3] The timing of the game is as follows. First, P offers contracts to A and B. Then, A and B decide whether to accept the offers. In case of acceptance, A and B simultaneously choose efforts. Finally, returns are realized and the agents receive their contracted payments.

3 Solution to the Basic Model

As there exist two binary returns R_A and R_B, all possible contracts for a single agent can be described by a tuple of four wages. Let w^i_{11} denote agent i's wage if both agents are successful and realize R, w^i_{10} (w^i_{01}) the wage of agent i if i succeeds and j fails (if i fails and j succeeds), and w^i_{00} agent i's wage if both fail ($i, j = A, B$; $i \neq j$). Given that both agents have accepted their contract offers $(w^i_{11}, w^i_{10}, w^i_{01}, w^i_{00})$, agent i chooses effort e_i to maximize expected utility

$$EU_i = e_i[e_j w^i_{11} + (1-e_j)\left(w^i_{10} + \alpha_i\right)] + (1-e_i)\left[e_j\left(w^i_{01} - \beta_i\right) + (1-e_j)\, w^i_{00}\right] - c_i(e_i).$$

As the objective function is strictly concave, the first-order condition describes i's incentive constraint:

$$e_j\left(\left[w^i_{11} - w^i_{10}\right] - \left[w^i_{01} - w^i_{00}\right] + \beta_i - \alpha_i\right) + \left[w^i_{10} - w^i_{00}\right] + \alpha_i = c'_i(e_i). \quad (1)$$

At the first stage of the game, P chooses the optimal contract $(w^{i*}_{11}, w^{i*}_{10}, w^{i*}_{01}, w^{i*}_{00})$ that maximizes

$$\begin{aligned} & Re_A + Re_B - e_A e_B\left(w^A_{11} + w^B_{11}\right) - e_A(1-e_B)\left(w^A_{10} + w^B_{01}\right) \\ & - (1-e_A)\, e_B\left(w^A_{01} + w^B_{10}\right) - (1-e_A)(1-e_B)\left(w^A_{00} + w^B_{00}\right), \end{aligned} \quad (2)$$

[3] E.g., if both agents have quadratic costs $\frac{\kappa}{2}e_i^2$, the condition yields $\kappa^2 > \Psi^2$.

subject to the incentive constraints (1) and the participation constraints $EU_i \geq 0$. Let $e^*_{i,UL}$ $(i = A, B)$ denote the efforts that are implemented by the optimal contract in the given setting with unlimited liability. In addition, let $\hat{e}^*_{i,UL}$ $(i = A, B)$ denote the optimal efforts from the same setting without peer effects. As the agents are risk neutral, in that case P would implement efforts that maximize material welfare $R \cdot e_A + R \cdot e_B - c_A(e_A) - c_B(e_B)$. Thus, $R = c'_i\left(\hat{e}^*_{i,UL}\right)$. Define $\bar{e}_i := (\beta_i - \alpha_j)/\Psi$ $(i, j = A, B;\ i \neq j)$.[4]

Proposition 1 *The following results hold for the optimal contract:*

(a) $e^*_{i,UL}$ *implemented by* $(w^{i*}_{11}, w^{i*}_{10}, w^{i*}_{01}, w^{i*}_{00})$ *is also implemented by the optimal contract that is only based on* R_i.

(b) *If* $\beta_i < \alpha_j$ *and* $\beta_j > \alpha_i$, *then* $e^*_{i,UL} < \hat{e}^*_{i,UL}$ *and* $e^*_{j,UL} > \hat{e}^*_{j,UL}$.

(c) *If* $\beta_i > \alpha_j$ *and* $\beta_j > \alpha_i$, *then* $e^*_{i,UL} \gtreqless \hat{e}^*_{i,UL}$ *iff* $e^*_{j,UL} \gtreqless \bar{e}_j$.

(d) *If* $\beta_i < \alpha_j$ *and* $\beta_j < \alpha_i$, *then* $e^*_{i,UL} \gtreqless \hat{e}^*_{i,UL}$ *iff* $e^*_{j,UL} \lesseqgtr \bar{e}_j$.

Result (a) shows that the optimal contract for agent i that makes wages contingent on both performance signals is equivalent to the optimal contract that compensates i only on the basis of R_i, although there exist externalities between the agents due to peer effects. In contrast to a situation without peer effects, the externalities lead to a game between A and B, which can be best seen from the incentive constraint that corresponds to the reduced contract (w^i_1, w^i_0), which pays agent i the wage w^i_1 (w^i_0) if $R_i = R$ $(R_i = 0)$:

$$\beta_i e_j + (1 - e_j)\alpha_i + \Delta w^i = c'_i(e_i) \qquad \text{with } \Delta w^i := w^i_1 - w^i_0. \tag{3}$$

Eq. (3) shows that incentives arise for three reasons. The wage spread Δw^i indicates standard textbook incentives. $(1 - e_j)\alpha_i$ characterizes i's additional incentives from peer effects conditional on j being unsuccessful – i

[4] All proofs are relegated to the appendix.

wants to benefit from positive externalities – and $\beta_i e_j$ additional peer incentives conditional on j being successful – i wants to avoid negative externalities. If $\beta_i > \alpha_i$ $(i = A, B)$, agents' efforts will be *strategic complements* in the notion of Bulow et al. (1985). Intuitively, if β_i is large, agent i's major aim is to prevent being outperformed by agent j. Hence, if j chooses high effort, i will also choose high effort to achieve at least a tie $R_j = R_i = R$, which would prevent β_i. If, on the contrary, j chooses low effort and most probably fails, then it is optimal for i to choose low effort as well since the probability of being outperformed by j is rather low and the extra utility from outperforming j, α_i, is not very large. If $\beta_i < \alpha_i$, then agents' efforts will be *strategic substitutes*. Now, outperforming j is attractive for i as α_i is large. Consequently, i's best response to a low value of e_j is a high value of e_i. On the other hand, a rather small β_i only leads to a low disutility for i when being outperformed by j so that the best response to a high value of e_j is a low value of e_i.

Technically, result (a) points out that the Informativeness Principle of Holmstrom (1979) applies despite the externalities between the agents. At first sight, this finding seems puzzling against the background of Englmaier and Wambach (2010), Weinschenk (2009), and Lang (2015). They show that, under a binding participation constraint, the principal prefers to violate the Informativeness Principle if the benefit of reduced inequity costs or ambiguity costs is sufficiently large. In the basic model of Section 2, the agents' participation constraints are also binding under the optimal contract so that the principal P directly suffers from negative externalities between the agents. However, sticking to the Informativeness Principle is always optimal for P – irrespective of the magnitude of the negative externalities. The intuition is the following. Consider the case of inequity aversion. As Englmaier and Wambach (2010) show, it can be optimal to choose team compensation for two agents that work on independent tasks because team compensation eliminates inequity costs stemming from income differences. Proposition 1 shows that optimal compensation is used to increase (decrease) the probability of positive (negative) externalities, but the Informativeness Principle is still valid because the *magnitude* of the externalities cannot be influenced by the compensation as the externalities directly flow into the agents' utility functions. Therefore, P sticks to individual performance to control incentives.

The three other results of Proposition 1 compare the efforts that are implemented by P in the presence of peer effects with those efforts that maximize material welfare and, hence, would be implemented without peer effects. Result (b) shows that if for one agent positive (negative) externalities from peer effects are larger than the negative (positive) externalities for the other agent, P will prefer excessive effort by the former one and little effort by the latter. The intuition is the following. Since agents are not protected by limited liability, the agents' participation constraints will be binding under the optimal contracts. As a consequence, P is the party that actually benefits from positive externalities and suffers from negative ones. If $\beta_i < \alpha_j$ and $\beta_j > \alpha_i$, P is mainly interested in agent j outperforming agent i and preventing i from outperforming j. Result (c) deals with the case where the negative externalities of each agent exceed the positive externalities of the respective other agent. In that situation, P has to solve a coordination problem to prevent that the agents' realized returns R_A and R_B differ. Either both agents should choose high efforts so that they both succeed with high probability, or both agents should choose low efforts so that both fail with high probability. Result (d) addresses the opposite case where the positive externalities of each agent exceed the negative externalities of the respective other agent. Now, P faces a reversed coordination problem: One of the agents should choose high effort and the other one low effort so that P realizes the net benefit $\alpha_A - \beta_B$ or $\alpha_B - \beta_A$, respectively.

When comparing P's expected profits with and without peer effects, the following result is obtained:

Proposition 2 *(a) If $\beta_i > (<) \alpha_j$ $(i, j = A, B;\ i \neq j)$, the principal will suffer (benefit) from peer effects. (b) If $\beta_i > \alpha_i$ $(i = A, B)$, the principal can still benefit from peer effects.*

Result (a) is straightforward. It directly follows from the agents' binding participation constraints. However, result (b) is more surprising. It shows that P can be better off by the existence of peer effects even if each agent's negative externalities exceed his positive externalities. This counterintuitive result can be explained as follows. Suppose that P wants to implement $e^*_{A,UL} < e^*_{B,UL}$. Hence, it is more likely that B instead of A is successful. If this effect sufficiently relaxes B's participation constraint because $\alpha_B - \alpha_A > 0$ is quite large, the overall effect on profits can be positive despite $\beta_i > \alpha_i$ $(i = A, B)$. Note that the constellation $\alpha_B > \beta_A > \alpha_A$ is feasible so that P will benefit from peer effects if the probability of B outperforming A is quite high and $\alpha_B - \beta_A$ is quite large.

4 Wealth-Constrained Agents

The basic model does not impose any restriction on feasible wages. This section considers the case that both agents are wealth-constrained so that wages are not allowed to be negative. Depending on the magnitude of the agents' externalities, the participation constraints may be binding or not in the optimum. In the following, I focus on the more interesting case where the constraints do not bind so that the agents earn positive rents. Let $e^*_{i,LL}$ $(i = A, B)$ denote the optimal efforts that are implemented by P in the given setting with limited liability and positive rents, and $\hat{e}^*_{i,LL}$ the corresponding optimal efforts in the same setting but without peer effects.[5] Let again $(w^{i*}_{11}, w^{i*}_{10}, w^{i*}_{01}, w^{i*}_{00})$ denote the optimal contracts chosen by P. The following results hold:

[5] In the absence of peer effects, agents always receive positive rents in equilibrium.

Proposition 3 *Suppose A and B are wealth-constrained and earn positive rents.*

(a) $e^*_{i,LL}$ *implemented by* $(w^{i*}_{11}, w^{i*}_{10}, w^{i*}_{01}, w^{i*}_{00})$ *is also implemented by the optimal contract that is only based on* R_i.

(b) $e^*_{i,LL}$ *and* $e^*_{j,LL}$ *are described by* $R + \Psi e^*_{j,LL} + \alpha_i = c'_i\left(e^*_{i,LL}\right) + e^*_i c''_i\left(e^*_{i,LL}\right)$ $(i, j = A, B; A \neq B)$. *If* $\Psi > 0$, *then* $e^*_{i,LL} > \hat{e}^*_{i,LL}$ *and* $\partial e^*_{i,LL}/\partial\Psi > 0$.

(c) $e^*_{i,LL} > e^*_{i,UL}$ *if and only if* $e^*_{i,UL} c''_i\left(e^*_{i,UL}\right) < \beta_j$ $(i, j = A, B; A \neq B)$.

(d) *P strictly benefits from peer effects.*

Result (a) shows that the Informativeness Principle is also valid in the case of limited liability and positive rents, that is, P does not want to manipulate the agents' rents by deviating from the major principle for creating incentives.

In the given situation, the principal is primarily interested in the incentive properties of peer effects, because the participation constraint is non-binding. P will strictly profit if peer effects boost incentives, since he does not have to compensate the agents for the negative externalities β_i, which only reduce agents' rents. The incentive constraints (3) show that, for given wages, both kinds of externalities increase agents' incentives – each agent i chooses high effort to benefit from positive externalities α_i and to avoid negative externalities β_i, which explains result (d).

Result (b) emphasizes the importance of Ψ in the given situation. If $\Psi > 0$, P will implement higher efforts with peer effects than without and optimal efforts will be increasing with Ψ. The intuition for both findings can be best explained by considering the incentive constraints (3). According to (3), efforts will be strategic complements if $\beta_i > \alpha_i$. P benefits from strategic complements, because incentivizing one agent leads to additional incentives for his peer and because these additional incentives are free for P, as argued in the paragraph before. Note that efforts being strategic complements (i.e.,

$\beta_i > \alpha_i$) is a sufficient condition for $\Psi := \beta_A + \beta_B - \alpha_A - \alpha_B > 0$, which completes the intuition.

Result (c) is presumably most interesting. It compares optimal efforts under unlimited and limited liability in the presence of peer effects. In case of unlimited liability, P implements efforts that maximize first-best welfare including expected utilities and disutilities from received externalities (see (8)). Qualitatively the same result is well-known from the principal-agent textbook model without peer effects as agents are risk neutral. From the textbook model we also know that the introduction of limited liability that leads to a positive rent for the agent must result into *inefficiently small* effort as the agent's rent increases in the implemented effort level. At first sight, a similar finding should also hold for the setting with peer effects. P now maximizes first-best welfare including expected utilities and disutilities from externalities minus the two agents' rents (see (13)), and the rent of each agent increases in his effort level. However, in case of peer effects, agent i's rent is given by[6]

$$r_i(e_i, e_j) := e_i c_i'(e_i) - c_i(e_i) - e_j \beta_i,$$

which differs from the rent in the textbook model by the term $-e_j \beta_i$. Thus, P faces a new trade-off when deciding on effort implementation: agent j's rent increases with e_j, but agent i's rent decreases with e_j. According to result (c), P will prefer *inefficiently large* effort implementation $e^*_{j,LL} > e^*_{j,UL}$ if i's rent reduction exceeds j's rent increase. The higher i's negative externality, β_i, the more this condition tends to be satisfied.

5 Sequential Moves

In the basic model, agents are assumed to choose efforts simultaneously. In this section, I keep the unlimited liability assumption of the basic model but

[6] $r_i(e_i, e_j)$ is obtained by combining i's objective function (10) with the incentive constraint.

assume that agents move sequentially. Let, agent A be the first mover and agent B the follower, i.e., first A chooses e_A and the two agents and P observe R_A. Thereafter, B chooses e_B and R_B is realized. As R_A is verifiable, P can make B's payment contingent on A's performance. Consequently, I consider two sorts of contracts. A is offered $\left(w_1^A, w_0^A\right)$, but B gets the contract offer $\left(w_0^B\left(R_A\right), w_1^B\left(R_A\right)\right)$ with wage spread $\Delta w^B\left(R_A\right) := w_1^B\left(R_A\right) - w_0^B\left(R_A\right)$. The game is solved backwards. If $R_A = R$, then B maximizes

$$w_1^B(R)\, e_B + \left(w_0^B(R) - \beta_B\right)(1 - e_B) - c_B(e_B),$$

and if $R_A = 0$, B maximizes

$$\left(w_1^B(0) + \alpha_B\right) e_B + w_0^B(0)(1 - e_B) - c_B(e_B).$$

Therefore, B's optimal effort is

$$e_B(R_A) = \begin{cases} p_B\left(\Delta w^B(R) + \beta_B\right) & \text{if} \quad R_A = R \\ p_B\left(\Delta w^B(0) + \alpha_B\right) & \text{if} \quad R_A = 0 \end{cases} \tag{4}$$

with p_B denoting the inverse of the marginal cost function c_B'. Agent A anticipates $e_B(R_A)$ and maximizes

$$\begin{aligned} &e_A p_B\left(\Delta w^B(R) + \beta_B\right) w_1^A + e_A\left[1 - p_B\left(\Delta w^B(R) + \beta_B\right)\right]\left(w_1^A + \alpha_A\right) - c_A(e_A) \\ &+ (1 - e_A)\, p_B\left(\Delta w^B(0) + \alpha_B\right)\left(w_0^A - \beta_A\right) + (1 - e_A)\left[1 - p_B\left(\Delta w^B(0) + \alpha_B\right)\right] w_0^A. \end{aligned}$$

Thus, agent A's optimal effort, e_A, is implicitly described by

$$\Delta w^A + \alpha_A + p_B\left(\Delta w^B(0) + \alpha_B\right)\beta_A - p_B\left(\Delta w^B(R) + \beta_B\right)\alpha_A = c_A'(e_A). \tag{5}$$

P anticipates $e_B(R_A)$ and e_A, and chooses the optimal contracts. Comparing expected profits in the simultaneous-move setting with those in the sequential-move setting leads to a clear-cut result:[7]

Proposition 4 *If P can choose between a simultaneous-move and a sequential-move setting, he will strictly prefer the latter one.*

The proof of Proposition 4 shows that, when agents move sequentially, P could implement the same efforts as in the simultaneous-move setting, but he strictly prefers other effort levels: If $e^*_{B,UL}$ denotes B's optimal effort in the simultaneous-move case, in the sequential-move setting P will implement $e_B(R)$ and $e_B(0)$ with

$$e_B(R) \gtreqless e^*_{B,UL} \gtreqless e_B(0) \Leftrightarrow \beta_B - \alpha_A \gtreqless \alpha_B - \beta_A.$$

Intuitively, P benefits from the fact that he can choose state-dependent incentives via $\Delta w^B(R)$ and $\Delta w^B(0)$. As an example, suppose that $\beta_B - \alpha_A > \alpha_B - \beta_A > 0$, which corresponds to the constellation $e_B(R) > e^*_{B,UL} > e_B(0)$. Thus, B's peer effects are stronger than A's so that – due to the binding participation constraints – P's relative loss from the negative externalities received by B (i.e., $\beta_B - \alpha_A$) exceeds P's relative gain from positive externalities received by B (i.e., $\alpha_B - \beta_A$). In this situation, it is most important for P to avoid an outcome where A succeeds and B fails. Therefore, if A is successful ($R_A = R$), then $e_B(R)$ should be very high and larger than B's effort given that A has failed, $e_B(0)$.

Appendix

Proof of Proposition 1. (a) Objective function (2) can be rewritten as

$$\begin{aligned}
&Re_A + Re_B \qquad (6)\\
&- e_A e_B \left[w^A_{11} - w^A_{10}\right] - (1-e_A)\, e_B \left[w^A_{01} - w^A_{00}\right] - e_A \left(w^A_{10} - w^A_{00}\right) - w^A_{00}\\
&- e_B e_A \left[w^B_{11} - w^B_{10}\right] - (1-e_B)\, e_A \left[w^B_{01} - w^B_{00}\right] - e_B \left(w^B_{10} - w^B_{00}\right) - w^B_{00}
\end{aligned}$$

[7] The proof is relegated to the additional pages for the referees.

and i's participation constraint, $EU_i \geq 0$, as

$$e_i e_j \left[w_{11}^i - w_{10}^i\right] + (1-e_i) e_j \left[w_{01}^i - w_{00}^i\right] + e_i \left(w_{10}^i - w_{00}^i\right) + w_{00}^i$$
$$+ e_i (1-e_j) \alpha_i - (1-e_i) e_j \beta_i - c_i(e_i) \geq 0. \qquad (7)$$

Hence, P maximizes (6) subject to (1) and (7). As the agents are not protected by limited liability, P chooses $[w_{11}^i - w_{10}^i]$, $[w_{01}^i - w_{00}^i]$, and $[w_{10}^i - w_{00}^i]$ to induce optimal incentives and w_{00}^i to extract all rents of the agents. Inserting the binding participation constraints (7) into (6) yields

$$\Pi_{UL}(e_A, e_B) := \qquad (8)$$
$$Re_A + Re_B + e_A e_B \Psi + e_A(\alpha_A - \beta_B) + e_B(\alpha_B - \beta_A) - c_A(e_A) - c_B(e_B).$$

Because the technical assumption $\inf_{e_A,e_B} c_A''(e_A) c_B''(e_B) > \Psi^2$ guarantees that P's second-order conditions $-c_i''\left(e_{i,UL}^*\right) < 0$ and $c_A''\left(e_{A,UL}^*\right) \cdot c_B''\left(e_{B,UL}^*\right) > \Psi^2$ hold ($i = A, B$), optimal efforts implemented by $(w_{11}^{i*}, w_{10}^{i*}, w_{01}^{i*}, w_{00}^{i*})$ are described by P's first-order conditions

$$R + e_{j,UL}^* \Psi + \alpha_i - \beta_j = c_i'\left(e_{i,UL}^*\right) \qquad (i, j = A, B;\, i \neq j). \qquad (9)$$

Suppose P offers agent i a contract (w_1^i, w_0^i) that is only based on R_i with wage w_1^i (w_0^i) being paid to i in case of $R_i = R$ ($R_i = 0$). Given that both agents have accepted their contract offers, i ($i = A, B$) maximizes

$$e_i[e_j w_1^i + (1-e_j)\left(w_1^i + \alpha_i\right)] + (1-e_i)\left[e_j\left(w_0^i - \beta_i\right) + (1-e_j) w_0^i\right] - c_i(e_i)$$
$$= e_i e_j \beta_i + e_i(1-e_j)\alpha_i + e_i \Delta w^i + w_0^i - e_j \beta_i - c_i(e_i) \qquad (10)$$

with $\Delta w^i := w_1^i - w_0^i$, leading to the incentive constraints $\beta_i e_j + (1-e_j)\alpha_i + \Delta w^i = c_i'(e_i)$. At the contracting stage, P maximizes expected profits $Re_A + Re_B - e_A \Delta w^A - w_0^A - e_B \Delta w^B - w_0^B$ subject to the incentive constraints and the participation constraints that (10) is non-negative. Due to unlimited liability, the agents' participation constraints are binding under the optimal contract (w_1^{i*}, w_0^{i*}). Inserting the binding participation constraints into $Re_A + Re_B - e_A \Delta w^A - w_0^A - e_B \Delta w^B - w_0^B$ leads to objective function (8). Thus, contracts $(w_{11}^{i*}, w_{10}^{i*}, w_{01}^{i*}, w_{00}^{i*})$ and (w_1^{i*}, w_0^{i*}) implement the same optimal efforts $e_{i,UL}^*$, described by (9), which proves result (a).

(b) Recall that $\hat{e}_{i,UL}^*$ is defined by $R = c_i' \left(\hat{e}_{i,UL}^*\right)$. If $\beta_i < \alpha_j$ and $\beta_j > \alpha_i$, then $e_{j,UL}^* \Psi + \alpha_i - \beta_j = -e_{j,UL}^* (\alpha_j - \beta_i) - (1 - e_{j,UL}^*)(\beta_j - \alpha_i) < 0$ and $e_{i,UL}^* \Psi + \alpha_j - \beta_i = e_{i,UL}^* (\beta_j - \alpha_i) + \left(1 - e_{i,UL}^*\right)(\alpha_j - \beta_i) > 0$ so that – according to (9) – $e_{i,UL}^* < \hat{e}_{i,UL}^*$ and $e_{j,UL}^* > \hat{e}_{j,UL}^*$.

(c) Let $\beta_i > \alpha_j$ and $\beta_j > \alpha_i$. Then, $\Psi > 0$, and $e_{i,UL}^* \gtreqless \hat{e}_{i,UL}^*$ if and only if $e_{j,UL}^* \Psi + \alpha_i - \beta_j \gtreqless 0 \Leftrightarrow e_{j,UL}^* \gtreqless (\beta_j - \alpha_i)/\Psi$.

(d) Let $\beta_i < \alpha_j$ and $\beta_j < \alpha_i$. Then, $\Psi < 0$, and $e_{i,UL}^* \gtreqless \hat{e}_{i,UL}^*$ if and only if $e_{j,UL}^* \Psi + \alpha_i - \beta_j \gtreqless 0 \Leftrightarrow e_{j,UL}^* \lesseqgtr (\beta_j - \alpha_i)/\Psi$.

Proof of Proposition 2. Consider the reduced contracts (w_1^i, w_0^i). Under peer effects, P can implement the same effort levels as in the situation without peer effects – i.e., $\hat{e}_{A,UL}^*$ and $\hat{e}_{B,UL}^*$ – by choosing Δw^A and Δw^B appropriately so that $\beta_i e_j + (1 - e_j)\alpha_i + \Delta w^i = R$ for $i, j = A, B$; $i \neq j$ in (3). Then, (8) shows that P will prefer (dislike) peer effects if

$$\hat{e}_{A,UL}^* \hat{e}_{B,UL}^* \Psi + \hat{e}_{A,UL}^* (\alpha_A - \beta_B) + \hat{e}_{B,UL}^* (\alpha_B - \beta_A) > (<) 0 \Leftrightarrow$$
$$\hat{e}_{A,UL}^* \left(1 - \hat{e}_{B,UL}^*\right)(\alpha_A - \beta_B) + \hat{e}_{B,UL}^* \left(1 - \hat{e}_{A,UL}^*\right)(\alpha_B - \beta_A) > (<) 0,$$

which proves result (a).

Now, consider (b). Define $\Delta_A := \beta_A - \alpha_A > 0$ and $\Delta_B := \beta_B - \alpha_B > 0$, and suppose that P again implements $\hat{e}_{A,UL}^*$ and $\hat{e}_{B,UL}^*$. Then, P will profit from peer effects if

$$\hat{e}^*_{A,UL}\hat{e}^*_{B,UL}\Psi + \hat{e}^*_{A,UL}(\alpha_A - \beta_B) + \hat{e}^*_{B,UL}(\alpha_B - \beta_A) > 0 \Leftrightarrow$$
$$-\left(1 - \hat{e}^*_{A,UL}\right)\hat{e}^*_{B,UL}\Delta_A - \left(1 - \hat{e}^*_{B,UL}\right)\hat{e}^*_{A,UL}\Delta_B + \left(\hat{e}^*_{B,UL} - \hat{e}^*_{A,UL}\right)(\alpha_B - \alpha_A) > 0.$$

Let, w.l.o.g., c_A be steeper than c_B implying $\hat{e}^*_{B,UL} > \hat{e}^*_{A,UL}$. Furthermore, let $\alpha_B - \alpha_A > 0$. Then, there exist upper bounds $\bar{\Delta}_i$ $(i = A, B)$ so that $\left(\hat{e}^*_{B,UL} - \hat{e}^*_{A,UL}\right)(\alpha_B - \alpha_A) > \left(1 - \hat{e}^*_{A,UL}\right)\hat{e}^*_{B,UL}\Delta_A + \left(1 - \hat{e}^*_{B,UL}\right)\hat{e}^*_{A,UL}\Delta_B$ for all $\Delta_i < \bar{\Delta}_i$ $(i = A, B)$.

Proof of Proposition 3. (a), (b) If P offers contracts $(w^i_{11}, w^i_{10}, w^i_{01}, w^i_{00})$ $(i = A, B)$ and the participation constraints are non-binding, he will maximize (6) subject to the agents' incentive constraints (1), which can be rewritten as

$$e_j w^i_{11} + e_j(\beta_i - \alpha_i) + (1 - e_j) w^i_{10} - (1 - e_j) w^i_{00} - e_j w^i_{01} + \alpha_i = c'_i(e_i).$$

Obviously, $w^i_{00} = w^i_{01} = 0$ is optimal to maximize incentives and reduce implementation costs. Thus, the incentive constraints simplify to $e_j w^i_{11} + (1 - e_j) w^i_{10} = c'_i(e_i) - e_j(\beta_i - \alpha_i) - \alpha_i$. Using this equation – together with $w^i_{00} = w^i_{01} = 0$ – (6) can be rewritten as

$$\Pi_{LL}(e_A, e_B) := \tag{11}$$
$$e_A\left[R + \beta_A e_B + (1 - e_B)\alpha_A - c'_A(e_A)\right] + e_B\left[R + \beta_B e_A + (1 - e_A)\alpha_B - c'_B(e_B)\right].$$

The first-order conditions

$$R + \Psi e^*_{j,LL} + \alpha_i = c'_i\left(e^*_{i,LL}\right) + e^*_{i,LL} c''_i\left(e^*_{i,LL}\right) \qquad (i, j = A, B;\ A \neq B) \tag{12}$$

will describe the optimal effort levels, if the second-order conditions hold. These are given by $-2c''_i\left(e^*_{i,LL}\right) - e^*_{i,LL}c'''_i\left(e^*_{i,LL}\right) < 0$ and $(2c''_A\left(e^*_{A,LL}\right) +$

$e^*_{A,LL} c'''_A \left(e^*_{A,LL}\right))(2c''_B \left(e^*_{B,LL}\right) + e^*_{B,LL} c'''_B \left(e^*_{B,LL}\right)) > \Psi^2$, where the last condition holds due to the technical assumptions $\inf_{e_A, e_B} c''_A (e_A)\, c''_B (e_B) > \Psi^2$ and $c'''_i (e_i) \geq 0$.

Suppose P offers agent i the contract (w^i_1, w^i_0), considered in the proof of Proposition 1. In the given situation with $w^i_0, w^i_1 \geq 0$ and positive rents, $w^i_0 = 0$ is optimal so that P maximizes $e_A(R - w^A_1) + e_B(R - w^B_1)$ subject to the incentive constraints $\beta_i e_j + (1 - e_j)\,\alpha_i + w^i_1 = c'_i (e_i)$. Solving for w^i_1 and inserting into $e_A(R - w^A_1) + e_B(R - w^B_1)$ shows that P maximizes (11).

As $\Psi, \alpha_i > 0$ and $\hat{e}^*_{i,LL}$ is described by $R = c'_i \left(\hat{e}^*_{i,LL}\right) + \hat{e}^*_{i,LL} c''_i \left(\hat{e}^*_{i,LL}\right)$, Eq. (12) immediately shows that $e^*_{i,LL} > \hat{e}^*_{i,LL}$.

Optimal efforts $e^*_{i,LL}$ are described by (12). Define the system of equations $F^i := R + \Psi e^*_{j,LL} + \alpha_i - \Omega\left(e^*_{i,LL}\right)$ with $\Omega\left(e^*_{i,LL}\right) := c'_i \left(e^*_{i,LL}\right) + e^*_{i,LL} c''_i \left(e^*_{i,LL}\right)$ $(i, j = A, B; A \neq B)$ for doing comparative statics via the implicit-function theorem. The corresponding Jacobian determinant

$$|J| = \begin{vmatrix} \frac{\partial F^A}{\partial e^*_{A,LL}} & \frac{\partial F^A}{\partial e^*_{B,LL}} \\ \frac{\partial F^B}{\partial e^*_{A,LL}} & \frac{\partial F^B}{\partial e^*_{B,LL}} \end{vmatrix} = \begin{vmatrix} -\Omega'\left(e^*_{A,LL}\right) & \Psi \\ \Psi & -\Omega'\left(e^*_{B,LL}\right) \end{vmatrix} = \Omega'\left(e^*_{A,LL}\right)\Omega'\left(e^*_{B,LL}\right) - \Psi^2$$

is positive as we know from the proof of result (b). Then, given $\Psi > 0$,

$$\frac{\partial e^*_{A,LL}}{\partial \Psi} = \frac{1}{|J|}\begin{vmatrix} -\frac{\partial F^A}{\partial \Psi} & \frac{\partial F^A}{\partial e^*_{B,LL}} \\ -\frac{\partial F^B}{\partial \Psi} & \frac{\partial F^B}{\partial e^*_{B,LL}} \end{vmatrix} = \frac{\Omega'\left(e^*_{B,LL}\right) e^*_{B,LL} + e^*_{A,LL}\Psi}{|J|} > 0.$$

Analogously, $\frac{\partial e^*_{B,LL}}{\partial \Psi} = [\Omega'\left(e^*_{A,LL}\right) e^*_{A,LL} + e^*_{B,LL}\Psi] / |J| > 0$.

(c) Define $r_i (e_i, e_j) := e_i c'_i (e_i) - c_i (e_i) - e_j \beta_i$. Then P's objective function $\Pi_{LL} (e_A, e_B)$ (see (11)) can be rewritten as

$$\Pi_{LL} (e_A, e_B) = \Pi_{UL} (e_A, e_B) - r_A (e_A, e_B) - r_B (e_B, e_A), \tag{13}$$

which is strictly concave because $\Pi_{UL} (e_A, e_B)$ describes P's objective function under unlimited liability of the agents (see (8)), which has the solution $\left(e^*_{A,UL}, e^*_{B,UL}\right)$. We will have $e^*_{i,LL} > e^*_{i,UL}$ if and only if $\frac{\partial}{\partial e_i}\Pi_{LL}\left(e^*_{A,UL}, e^*_{B,UL}\right) > 0$. As $\frac{\partial}{\partial e_i}\Pi_{UL}\left(e^*_{A,UL}, e^*_{B,UL}\right) = 0$, we obtain

$$\frac{\partial}{\partial e_i}\Pi_{LL}\left(e^*_{A,UL}, e^*_{B,UL}\right) > 0 \Leftrightarrow e^*_{i,UL}c''_i\left(e^*_{i,UL}\right) < \beta_j.$$

(d) In principle, P could implement the same effort levels as in the situation without peer effects. From (11) we can see that P then unambiguously benefits from peer effects, since $\beta_A e_A e_B + e_A\left(1 - e_B\right)\alpha_A + \beta_B e_A e_B + e_B\left(1 - e_A\right)\alpha_B > 0$.

References

Bolton, G.E. and A. Ockenfels (2000): ERC: A Theory of Equity, Reciprocity, and Competition. American Economic Review 90, 166–193.

Bulow, J.I., Geanakoplos, J.D. and P.D. Klemperer (1985): Multimarket Oligopoly: Strategic Substitutes and Complements. Journal of Political Economy 93, 488–511.

Charness, G. and P. Kuhn (2011): Lab Labor: What Can Labor Economists Learn from the Lab? Handbook of Labor Economics, Volume 4a, 229–330.

Charness, G., Masclet, D., and M.C. Villeval (2014): The Dark Side of Competition for Status. Management Science 60, 38–55.

Charness G. and M. Rabin (2002): Understanding Social Preferences with Simple Tests. Quarterly Journal of Economics 117, 817–869.

Dohmen, T., Falk, A., Fliessbach, K., Sunde, U. and B. Weber (2011): Relative versus Absolute Income, Joy of Winning, and Gender: Brain Imaging Evidence. Journal of Public Economics 95, 279–285.

Englmaier, F. and A. Wambach (2010): Optimal Incentive Contracts Under Inequity Aversion. Games and Economic Behavior 69, 312–328.

Falk, A. and A. Ichino (2006): Clean Evidence on Peer Effects. Journal of Labor Economics 24, 39–58.

Fehr, E. and K. Schmidt (1999): A Theory of Fairness, Competition and Co-operation. Quarterly Journal of Economics 114, 817–868.

Fershtman, C., Hvide, H.K., and Y. Weiss (2003): A Behavioral Explanation of the Relative Performance Evaluation Puzzle. Annales D' Économie et de Statistique No. 71/72, 349–361.

Gächter, S., Nosenzo, D. and M. Sefton (2013): Peer Effects in Pro-Social Behavior: Social Norms or Social Preferences? Journal of the European Economic Association 11, 548–573.

Georganas, S., Tonin, M. and M. Vlassopoulos (2013): Peer Pressure and Productivity: The Role of Observing and Being Observed. IZA Discussion Paper No. 7523.

Holmstrom, B. (1979): Moral Hazard and Observability. Bell Journal of Economics 10, 74–91.

Holmstrom, B. (1982): Moral Hazard in Teams. Bell Journal of Economics 13, 324–340.

Kräkel, M. (2008a): Emotions in Tournaments. Journal of Economic Behavior and Organization 67, 204–214.

Kräkel, M. (2008b): Emotions and Compensation. Schmalenbach Business Review 60, 145–159.

Lang, M. (2015): First-Order and Second-Order Ambiguity Aversion. Mimeo.

Malcomson, J.M. (1984): Work Incentives, Hierarchy, and Internal Labor Markets. Journal of Political Economy 92, 486–507.

Mas, A. and E. Moretti (2009): Peers at Work. American Economic Review 99, 112–145.

Moldovanu, B. and A. Sela (2001): The Optimal Allocation of Prizes in Contests. American Economic Review 91, 542–558.

Ohlendorf, S. and P.W. Schmitz (2012): Repeated Moral Hazard and Contracts with Memory: The Case of Risk-Neutrality. International Economic Review 53, 433–452.

Schmitz, P.W. (2005): Workplace Surveillance, Privacy Protection, and Efficiency Wages. Labour Economics 12, 727–738.

Sheremeta, R.M. (2010): Experimental Comparison of Multi-Stage and One-Stage Contests. Games and Economic Behavior 68, 731–747.

Weinschenk, P. (2009): Essays in Microeconomics. Dissertation. University of Bonn.

Winter, E. (2010): Transparency and Incentives Among Peers. RAND Journal of Economics 41, 504–523.

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Proof of Proposition 4:

Similar to the basic model with reduced contracts (w_1^i, w_0^i), the agents' participation constraints will be binding under the optimal contracts:

$$e_A\left[e_B\left(0\right)\beta_A - e_B\left(R\right)\alpha_A\right] + e_A\alpha_A - e_B\left(0\right)\beta_A - c_A\left(e_A\right) = -e_A\Delta w^A - w_0^A$$

and

$$e_A\left[e_B\left(R\right)\beta_B - e_B\left(0\right)\alpha_B\right] + e_B\left(0\right)\alpha_B - e_A\beta_B - e_A c_B\left(e_B\left(R\right)\right) - \left(1-e_A\right)c_B\left(e_B\left(0\right)\right)$$
$$= -e_A e_B\left(R\right)\Delta w^B\left(R\right) - \left(1-e_A\right)e_B\left(0\right)\Delta w^B\left(0\right) - \left(1-e_A\right)w_0^B\left(0\right) - e_A w_0^B\left(R\right)$$

with $e_B\left(0\right) = p_B\left(\Delta w^B\left(0\right) + \alpha_B\right)$ and $e_B\left(R\right) = p_B\left(\Delta w^B\left(R\right) + \beta_B\right)$. Inserting the binding constraints in P's objective function,

$$\begin{aligned}
&R\left[e_A + e_A e_B\left(R\right) + \left(1-e_A\right)e_B\left(0\right)\right] - e_A w_1^A - \left(1-e_A\right)w_0^A \\
&- e_A e_B\left(R\right)w_1^B\left(R\right) - \left(1-e_A\right)e_B\left(0\right)w_1^B\left(0\right) \\
&- e_A\left(1-e_B\left(R\right)\right)w_0^B\left(R\right) - \left(1-e_A\right)\left(1-e_B\left(0\right)\right)w_0^B\left(0\right),
\end{aligned}$$

yields

$$\begin{aligned}
&R\left[e_A + e_A e_B\left(R\right) + \left(1-e_A\right)e_B\left(0\right)\right] + e_A\left(\alpha_A - \beta_B\right) + e_B\left(0\right)\left[\alpha_B - \beta_A\right] \\
&+ e_A\left[e_B\left(0\right)\beta_A + e_B\left(R\right)\beta_B - e_B\left(R\right)\alpha_A - e_B\left(0\right)\alpha_B\right] \qquad (14) \\
&- c_A\left(e_A\right) - e_A c_B\left(e_B\left(R\right)\right) - \left(1-e_A\right)c_B\left(e_B\left(0\right)\right).
\end{aligned}$$

P can implement any efforts he likes by appropriately choosing the wage spreads $\Delta w^B\left(R\right)$, $\Delta w^B\left(0\right)$, and Δw^A in the incentive constraints (4) and (5). In particular, P can implement $e_B\left(R\right) = e_B\left(0\right)$ so that his objective functions for the simultaneous-move setting (i.e., (8)) and the sequential-move setting (i.e., (14)) coincide. Thus, letting agents move sequentially instead of simultaneously cannot be detrimental for P.

However, it can be shown that P *strictly* prefers $e_B(R) \neq e_B(0)$, implying that P is better off in the sequential-move setting: The first-order conditions for the optimal efforts $e_B^*(R)$ and $e_B^*(0)$ lead to a unique solution being implicitly described by $R + \beta_B - \alpha_A = c_B'(e_B^*(R))$ and $R + \alpha_B - \beta_A = c_B'(e_B^*(0))$. Hence, $e_B^*(R) \neq e_B^*(0)$ because $\beta_B - \alpha_A \neq \alpha_B - \beta_A \Leftrightarrow \Psi \neq 0$ is true.

As second-order condition, the Hessian matrix

$$\begin{bmatrix} \frac{\partial^2\Pi}{\partial e_A^2} & \frac{\partial^2\Pi}{\partial e_A \partial e_B(0)} & \frac{\partial^2\Pi}{\partial e_A \partial e_B(R)} \\ \frac{\partial^2\Pi}{\partial e_B(0)\partial e_A} & \frac{\partial^2\Pi}{\partial e_B^2(0)} & \frac{\partial^2\Pi}{\partial e_B(0)\partial e_B(R)} \\ \frac{\partial^2\Pi}{\partial e_B(R)\partial e_A} & \frac{\partial^2\Pi}{\partial e_B(R)\partial e_B(0)} & \frac{\partial^2\Pi}{\partial e_B^2(R)} \end{bmatrix} =$$

$$\begin{bmatrix} -c_A''(e_A) & -R+\beta_A-\alpha_B+c_B'(e_B(0)) & R+\beta_B-\alpha_A-c_B'(e_B(R)) \\ -R+\beta_A-\alpha_B+c_B'(e_B(0)) & -(1-e_A)\,c_B''(e_B(0)) & 0 \\ R+\beta_B-\alpha_A-c_B'(e_B(R)) & 0 & -e_A c_B''(e_B(R)) \end{bmatrix}$$

has to be negative definite. This will be the case, if the first principal minor is negative (which is true: $-c_A''(e_A) < 0$), the second principal minor is positive, i.e.,

$$\begin{vmatrix} -c_A''(e_A) & -R+\beta_A-\alpha_B+c_B'(e_B(0)) \\ -R+\beta_A-\alpha_B+c_B'(e_B(0)) & -(1-e_A)\,c_B''(e_B(0)) \end{vmatrix}$$
$$= c_A''(e_A)\,(1-e_A)\,c_B''(e_B(0)) - \left[-R+\beta_A-\alpha_B+c_B'(e_B(0))\right]^2 > 0,$$

which is true since $-R + \beta_A - \alpha_B + c_B'(e_B(0)) = 0$ must hold as first-order condition, and the third principal minor is negative, i.e.,

$$\begin{vmatrix} -c_A''(e_A) & -R+\beta_A-\alpha_B+c_B'(e_B(0)) & R+\beta_B-\alpha_A-c_B'(e_B(R)) \\ -R+\beta_A-\alpha_B+c_B'(e_B(0)) & -(1-e_A)\,c_B''(e_B(0)) & 0 \\ R+\beta_B-\alpha_A-c_B'(e_B(R)) & 0 & -e_A c_B''(e_B(R)) \end{vmatrix}$$

$$= -c_A''(e_A)\begin{vmatrix} -(1-e_A)\,c_B''(e_B(0)) & 0 \\ 0 & -e_A c_B''(e_B(R)) \end{vmatrix}$$
$$-\left(-R+\beta_A-\alpha_B+c_B'(e_B(0))\right)\begin{vmatrix} -R+\beta_A-\alpha_B+c_B'(e_B(0)) & 0 \\ R+\beta_B-\alpha_A-c_B'(e_B(R)) & -e_A c_B''(e_B(R)) \end{vmatrix}$$
$$+\left(R+\beta_B-\alpha_A-c_B'(e_B(R))\right)\begin{vmatrix} -R+\beta_A-\alpha_B+c_B'(e_B(0)) & -(1-e_A)\,c_B''(e_B(0)) \\ R+\beta_B-\alpha_A-c_B'(e_B(R)) & 0 \end{vmatrix}$$

$$\begin{aligned} &= -c_A''(e_A)\,(1-e_A)\,c_B''(e_B(0))\,e_A c_B''(e_B(R)) \\ &+\left(-R+\beta_A-\alpha_B+c_B'(e_B(0))\right)^2 e_A c_B''(e_B(R)) \\ &+\left(R+\beta_B-\alpha_A-c_B'(e_B(R))\right)^2 (1-e_A)\,c_B''(e_B(0)) < 0, \end{aligned}$$

which is true, because we have $-R+\beta_A-\alpha_B+c_B'(e_B(0))=0$ and $R+\beta_B-\alpha_A-c_B'(e_B(R))=0$ as first-order conditions.